

# The Interaction Of Multiple Convection Zones In A-type Stars

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1 February 2008

## ABSTRACT

A-type stars have a complex internal structure with the possibility of multiple convection zones. If not sufficiently separated, such zones will interact through the convectively stable regions that lie between them. It is therefore of interest to ask whether the typical conditions that exist within such stars are such that these convection zones can ever be considered as disjoint.

In this paper we present results from numerical simulations that help in understanding how increasing the distance between the convectively unstable regions affects the interaction. We go on to discuss the effect of varying the stiffness of the stable layer that lies between the unstable regions. We show that in A-type stars the convectively unstable regions are likely to interact through the stable region that separates them. This has profound implications for mixing and transport within these stars.

**Key words:** convection – stars:interior

## 1 INTRODUCTION

The internal structure of main sequence stars varies greatly according to their spectral type (Schwarzschild (1965)). For example, stars like the Sun typically have a radiative core with a superadiabatic, convecting outer region. On the other hand upper main sequence stars, including A-type stars, are frequently pictured to consist of a radiative outer layer that surrounds a large convecting hydrogen burning core. Such variation in structure is important as it can give rise to drastically different transport mixing rates of both passive and dynamic quantities within these stars. In addition, the connection to the stellar atmosphere, which lies above the photosphere, is fundamentally different if the outermost part of the interior is convectively stable or convectively unstable.

Interactions within A-type stars also lead to different chemical balances, which can give rise to not just one but multiple convection zones for main-sequence stars of this type. Observations indicate that, near to the surface of these stars there is at least one convection zone (Landstreet (1998); Silaj *et al.* (2005)). Further considerations of a theoretical nature predict at least one further convection zone near the surface (Toomre *et al.* (1976); Kupka (2005)). Such zones are believed to be thin (if compared to the radius of the star) yet important from the point of view of transportation and mixing. Their properties have been discussed for several

decades (Siedentopf (1933); Latour, Toomre & Zahn (1976); Kupka (2005)). The outer convection zone in these stars, immediately below the surface, is caused by the partial ionization of hydrogen and the single ionization of helium. The lower convection zone is the result of the second ionization of helium (Latour, Toomre & Zahn (1976); Kupka (2005)) and at least one further convection zone has been postulated (Kupka (2005)). There has been a long-standing debate about the nature or even existence of the inner convection zone(s); in particular whether it can be consistent with the gravitational settling of heavier elements (see for example the papers of Vauclair, Vauclair & Pamjatnikh (1974) and Richer, Michaud & Proffitt (1992)). We do not attempt to address these questions in the present study; rather, we focus on the basic problem; given that two layers can at least plausibly coexist, what factors influence their interaction, and to what extent do earlier simplified models give a correct picture of this interaction?

It is now well known that in stars, without the presence of rigid boundaries, ascending and descending convectively driven motions overshoot the layer that is convectively unstable. In the Sun (a G-type star) such overshooting is observed at the solar photosphere and is witnessed in the form of granulation. Further, such overshooting occurs at the bottom of the solar convection zone and is believed to play an important role in the solar dynamo because it transports magnetic field into the solar tachocline (Tobias *et al.* (1998); Tobias *et al.* (2001)); this is an important part of the interface dynamo model originally proposed by Parker

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(Parker (1993)). Such overshooting behaviour will naturally occur at other similar interfaces. When there are multiple convection zones such overshooting leads to enhanced communication and transport between the unstable layers. It is of importance to understand the nature of this interaction.

In A-type stars in particular, with two convection zones that are in quite close proximity, fascinating dynamics and mixing may occur. Overshooting plumes from an upper convection zone and a lower convection zone can interact in the convectively stable region separating them. Furthermore if conditions are right it is possible for plumes to overshoot completely and pierce the other convection zone, which would lead to transportation of ‘contaminants’ directly from one convectively unstable region into the other. Therefore, the two fundamental questions are: How far apart do these layers need to be before they can be considered as disjoint; and how close must they be for direct penetration from one layer to the other to occur.

Earlier analytic work (Toomre *et al.* (1976); Latour, Toomre & Zahn (1976)) on this problem adopted a mean-field approach, which gives a highly simplified view of the nonlinear interactions but allows the reduction of the problem to a relatively simple set of ODE’s, and it serves as a guide for the fully compressible simulations that are the subject of this paper. Latour *et al.* concluded that the two convectively unstable layers need to be separated by a distance of at least two pressure scale heights for there to be no interaction between the layers. Such a condition can be achieved in a number of ways via the variation of the different parameters that naturally occur in the model. One way is to increase the vertical extent of the domain and so increase the width of the intermediate layer. Another way is to vary the conductivity of the mid layer. In this paper we choose to explore the effects of both of these changes given that the separation of the two zones as well as their relative conductivities can be different in different stars. For simplicity we choose to focus on convective layers of fixed width but we note that one could alternatively fix the domain height and decrease the width of the convection zones and so increase the width of the convectively stable region; this was the approach adopted in a preliminary investigation by Muthsam, Wolfgang, Friedrich & Liebich (Muthsam *et al.* (1999)).

Muthsam *et al.* examined three cases in a three dimensional model in a Cartesian geometry with a small aspect ratio. In this simple study they showed that bringing the two convection zones closer together, by shrinking the width of the convectively unstable region, led to the convection layers merging as the interaction between the layers increased. However, their preliminary investigation warrants a more detailed study for a number of reasons. First, while they did make a passing remark as to the pressure scale height change across the box they did not comment on how the pressure scale height changes across the mid-layer, which Toomre *et al.* indicated was the important factor. Further, while they acknowledge the earlier work by Toomre *et al.* they did not relate their numerical calculation directly to that work.

This paper is organised as follows: In the next section we provide a detailed discussion of our model, the numerical method used to solve the equations and the parameters that we select. In section 3 we examine the effect of varying the

mid-layer thickness and stiffness of the convectively stable region before concluding in section 4.

## 2 MODEL

We consider the evolution of a compressible fluid in a layer and consider a model that is in the spirit of earlier papers on penetrative convection (Tobias *et al.* (1998); Tobias *et al.* (2001)); these in turn represent a simple extension of studies of convection in a single Cartesian layer. The following scalings are used to express the equations in dimensionless form (Matthews, Proctor, & Weiss (1995)): lengths are scaled with the layer depth  $d$ , times with the isothermal sound travel time  $d/\sqrt{R_*T_0}$ , density with its value at the top of the layer  $\rho_0$ , temperature with its value at the top of the layer  $T_0$ .

The governing equations can then be expressed as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{u} = 0, \quad (1)$$

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla P + \theta(m+1)\rho \hat{\mathbf{z}} + \sigma \kappa \nabla \cdot \rho \tau, \quad (2)$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = -(\gamma-1)T \nabla \cdot \mathbf{u} + \frac{\kappa(\gamma-1)\sigma \tau^2}{2} + \frac{\gamma \kappa}{\rho} \nabla^2 T, \quad (3)$$

where  $z$  is taken downward,  $\theta$  is the dimensionless temperature difference across the layer,  $R_*$  is the gas constant,  $m$  is the polytropic index,  $\kappa = K/d\rho_0 c_P \sqrt{R_*T_0}$  is the dimensionless thermal diffusivity,  $\gamma$  is the ratio of specific heats,  $\tau$  is the stress tensor given by:

$$\tau_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k}, \quad (4)$$

$P = \rho T$  and  $\sigma$  is the Prandtl number.

In order to achieve the required basic state we allow the thermal profile to be non-linear and we take

$$\begin{aligned} K &= \frac{K_1}{2} \left[ 1 + \frac{K_2 + K_3}{K_1} - \tanh \left( \frac{z-1}{\Delta} \right) \right. \\ &\quad + \frac{K_3}{K_1} \tanh \left( \frac{z-\mathcal{D}+1}{\Delta} \right) \\ &\quad \left. - \frac{K_2}{K_1} \tanh \left( \frac{z-\mathcal{D}+1}{\Delta} \right) \tanh \left( \frac{z-1}{\Delta} \right) \right] \end{aligned} \quad (5)$$

where  $\Delta$  is the characteristic size of the transition region between each of the layers. In this work the characteristic sizes of the transition regions are taken to be the same for simplicity. The static density and temperature profiles are found by solving the equations of hydrostatic balance. To this static state, throughout the domain, random perturbations are introduced, with amplitudes which lie within the interval  $[-0.05, 0.05]$ .

The aspect ratio for the computational domain in this study is  $4.4:\mathcal{D}$ , where  $\mathcal{D}$  is the total depth of the box, and the domain is assumed to be periodic in  $x$  and  $y$ . The conditions on the upper and lower boundaries are:

$$T = 1, \quad u_z = 0, \quad \frac{\partial u_x}{\partial z} = 0 \quad \text{at } z = 0. \quad (6)$$

$$\frac{\partial T}{\partial z} = \theta, \quad u_z = 0, \quad \frac{\partial u_x}{\partial z} = 0. \quad \text{at } z = \mathcal{D}. \quad (7)$$

The governing equations above are solved using a parallel hybrid finite-difference/pseudo-spectral code; the most

Param.	Description	Value
$\sigma$	Prandtl Number	1.0
$m_1 = m_3$	Polytropic Index in layers 1 and 3	1.0
$\theta$	Thermal Stratification	10
$\gamma$	Ratio of Specific Heats	5/3
$R_a$	Rayleigh number	$1.7 \times 10^5$

**Table 1.** The parameter definitions and values.

comprehensive description of the code can be found in (Matthews, Proctor, & Weiss (1995)). Nonlinear products are performed in configuration space, the transformation from phase space being facilitated by fast fourier transforms. Time-stepping is carried out in configuration space and an explicit second-order Adams-Bashforth scheme is used with variable weights to accommodate adaptive step-sizes.

The system we study has a large number of dimensionless parameters, making it impractical to conduct a complete survey. Thus a number are held fixed at values shown in Table 1. These parameters have been chosen so that we have time dependent, highly supercritical convection occurring in both convection zones. Of course, stellar convection operates in a highly turbulent regime. The computational cost of simulating fully turbulent convection at very high Reynolds number is presently prohibitive. Nonetheless our flows are fully time-dependent and possess sufficient spatial complexity for our purposes.

We use a subscript 1 on quantities relating to the upper convection zone. Similarly, subscript 2 will be used to denote quantities for the convectively stable layer and 3 to denote quantities in the lower convective zone.

The stiffness parameter,  $S$ , provides a useful measure of the relative conductivities in this problem (for more detailed discussion see, Hurlburt *et al.* (1994); Tobias *et al.* (1998)).  $S_2$  and  $S_3$  are related to the various polytropic indexes that appear in the problem via:

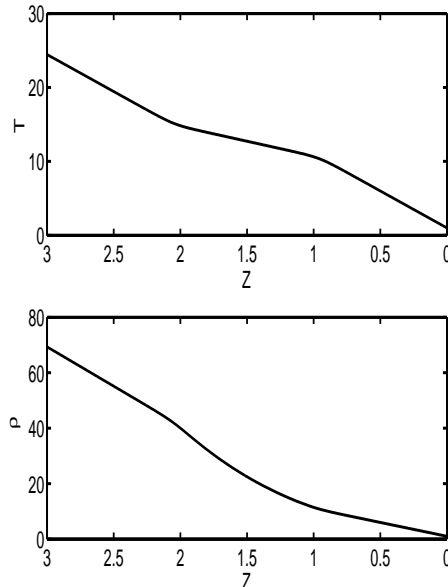
$$S_2 = \frac{m_2 - m_{ad}}{m_{ad} - m_1} \quad (8)$$

$$S_3 = \frac{m_3 - m_{ad}}{m_{ad} - m_1} \quad (9)$$

Since  $m_3 = m_1$ ,  $S_3 = S_1$  for this simple model.

### 3 THE EFFECT OF VARYING THE THICKNESS OF THE STABLE LAYER

The primary focus of this paper is to explore the effect of increasing the width of the convectively stable region, via increasing the total domain depth  $\mathcal{D}$ , on strength of the interaction between the unstable layers. Increasing the width of the convectively stable zone is the most natural way to increase the number of pressure scale heights across this region. We begin by considering the case where all three zones are equal and so  $\mathcal{D} = 3.0$ . The resolution for this case is 64:64:400. Our tests have shown, as is frequently the case in convection simulations, that fewer modes are needed in the  $x$  and  $y$  directions than grid points needed in the vertical. We employ this resolution throughout, except in the next section where the height of the box is varied.

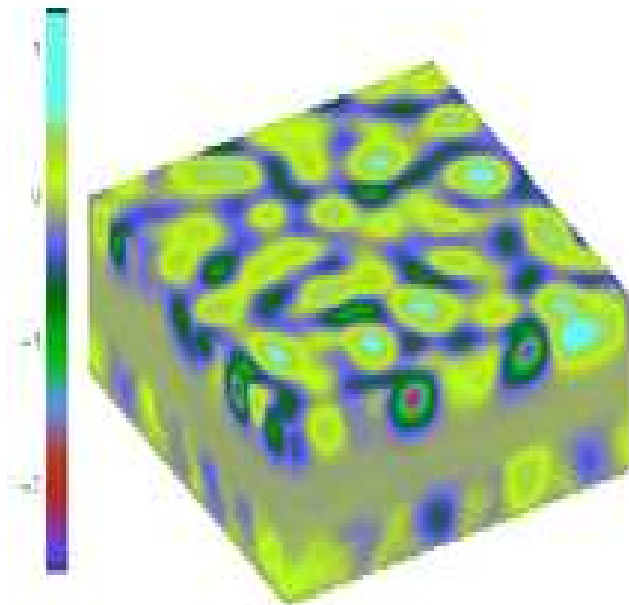


**Figure 1.** The initial temperature and density profiles when  $\mathcal{D} = 3.0$ .

As we outlined in the previous section, the conductivity is non-linear in this problem and so we must solve for the initial density and temperature profile for each case. For the fiducial case (for which the stable region is the same width as the unstable regions) the initial temperature and density variations are shown in Figure 1. As the earlier analytic theory indicated that the number of pressure scale heights of separation between the two unstable layers is a crucial factor we will focus on this quantity. For the fiducial case there are 1.60 pressure scale heights across the convectively stable mid-layer, which is rather less than the two pressure scale heights estimate suggested by the earlier analytic theory as necessary for true separation.

The state shown in Figure 1 is perturbed and allowed to evolve. At early times, as Figure 2 shows, the motion is strongest in the upper convectively unstable zone while comparatively small at the bottom of the box. However, Figure 3 shows that as time progresses motion in the lower layer becomes more vigorous. A statistically steady state is fully established within a few turnover times although there are significant temporal fluctuations. This state is illustrated in Figure 4 and Figure 5 shows horizontal slices through each of the three zones. As one might expect the convection is noticeably different in the two convection zones and we find that the average kinetic energy in the middle zone is comparable with that for the top region but that the energy in the bottom region is three times larger.

While Figure 4 provides a useful picture of the convective state it is difficult to gauge the motions within the convectively stable region from such a figure. To obtain a clearer picture of the potential interactions between the two convectively unstable regions it is helpful to note that, if the two layers are to be considered as ‘independent’ from each other, then there needs to be a region between the layers where the velocity becomes very small. Therefore, to facilitate a clearer picture of the degree of interaction of the layers



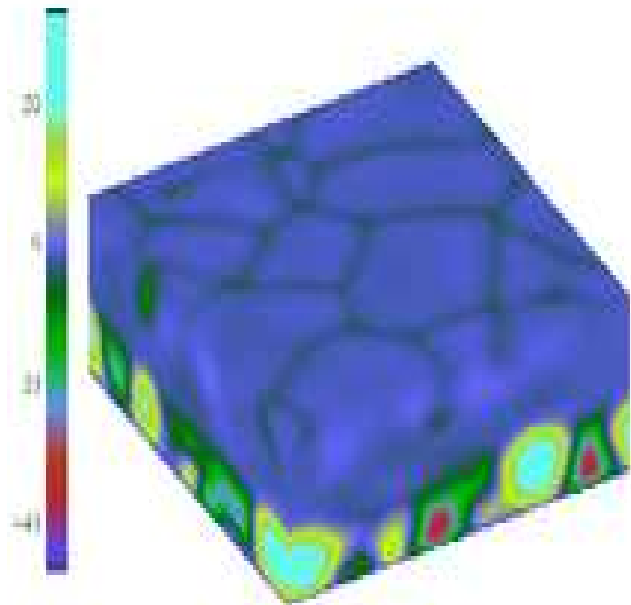
**Figure 2.** Plot at  $t = 2.81$  Sides of the box show the vertical momentum flux and the top shows the vertical momentum flux near the top of the box for the case where  $\mathcal{D} = 3.0$  and  $S_2 = 5.0$ .

we calculate the variation in  $z$  of the horizontal averages of the modulus of the  $z$  component of momentum.

Figure 7 shows that while there is clearly more vertical motion in the convectively unstable regions, as one might anticipate, there is still a non-negligible vertical component of momentum in the middle of the box. Thus the two convection zones are connected in this case and so there is a conduit for mixing between the two convection zones for this level of separation. Any reduction in the width of the convectively stable region will decrease the number of pressure scale heights of variation across the layer and increase the level of interaction between the two unstable zones. Such vigorous motion in the convectively stable layer was discussed in the context of the downward directed hexagon case of Latorre *et al.* and is further confirmed by a plot of the vertical component of velocity as shown in 6.

Although the Rayleigh number of the convection is very large the numerical resolution is not available to conduct simulations with very large Reynolds numbers; here the peak values are of order 100. Improving the resolution, and so using even larger Rayleigh numbers, would serve to increase the amount of interaction of the unstable layers.

The most obvious way to limit the interaction of the unstable regions is to increase the layer depth,  $\mathcal{D}$ , and so increase the width of the stable region if the convectively unstable regions have the same height. In this paper we discuss three further values of  $\mathcal{D}$  namely, 3.5, 4.0 and 5.0, for which the corresponding pressure scale height variations across the mid-layer are 2.23, 2.80 and 3.77. It is important to note that all three of these cases are above the two pressure scale



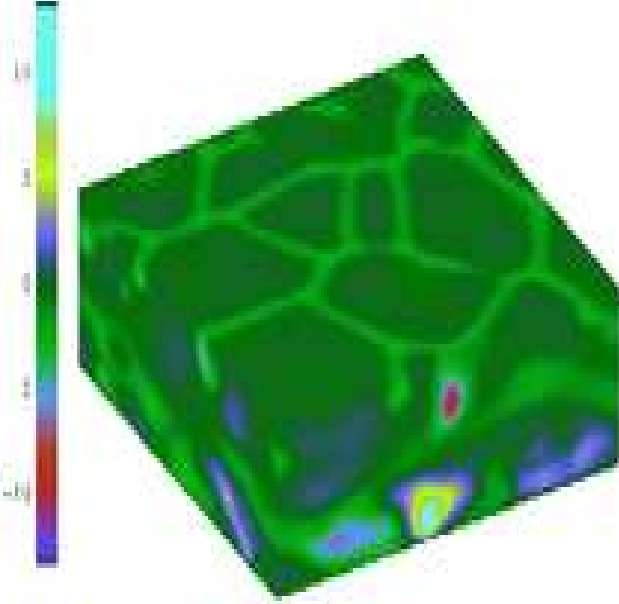
**Figure 3.** Plot at  $t = 8.41$  Sides of the box show the vertical momentum flux and the top shows the vertical momentum flux near the top of the box for the case where  $\mathcal{D} = 3.0$  and  $S_2 = 5.0$ .

heights estimate that suggested by the analytic theory as the transition between connected and unconnected convection layers.

Figure 8 shows a snapshot of the vertical component of the momentum for the case where  $\mathcal{D} = 3.5$ . This figure is qualitatively the same as that for  $\mathcal{D} = 3.0$  and we note here that similar plots are obtained at larger box heights.

As for the case when  $\mathcal{D} = 3.0$ , we consider the variation of the planar average vertical component of momentum throughout the domain in the established statistically steady state, for each of the increased box heights. Figures 9-11 show this quantity for each case. Note that there is a change in the magnitude of the vertical momentum flux in the bottom zone as the box height is increased. This is because the density in this lower region increases as the box height is elongated. This is a follow on effect of increasing the stable layer. One should note that in the static state the density and temperature differences across the upper convection zone remains unchanged. The increase in the depth of the convectively stable region alters the difference in these quantities across the stable layer, which implies a greater density at the top of the lower convection zone.

Each of these figures clearly illustrates that, while there is a significant reduction in vertical motion in the convectively stable region, there is a non negligible vertical component of momentum throughout the box for all box heights. The motion within the convectively stable region in all these cases is generated by overshooting plumes from the convectively unstable regions that lie on either side of this layer. As

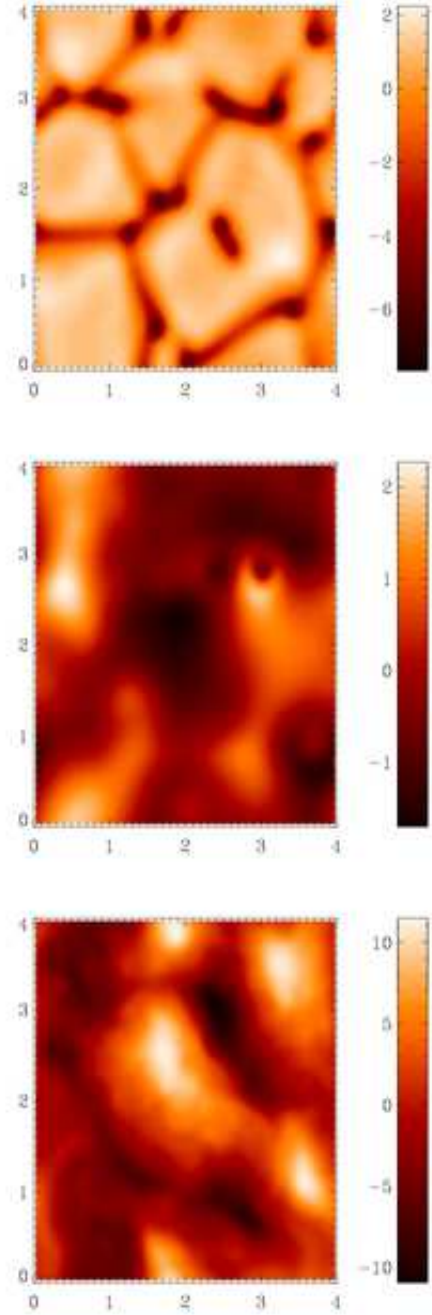


**Figure 4.** Plot at  $t = 15.02$  Sides of the box show the vertical momentum flux and the top of the box shows the vertical momentum flux near the top of the box for the case where  $\mathcal{D} = 3.0$  and  $S_2 = 5.0$ .

there is no point at which the vertical component of momentum vanishes in any of these cases there is a route for mixing of passive and dynamic quantities between the two convectively unstable regions. Only for a substantial increase in the box height will the vertical component of momentum fall to a very small value at one plane in the box. However, further increase in the box height would be extremely computationally expensive for this problem and also unwarranted in the physical context. In A-type main sequence stars the separation between the unstable layers does not extend past a couple of scale heights. Therefore, on the basis of the present work we can conclude that in A-type stars there is a clear connection between the convectively unstable zones that lie immediately below the stellar photosphere.

#### 4 THE EFFECT OF VARYING THE STIFFNESS OF THE STABLE LAYER

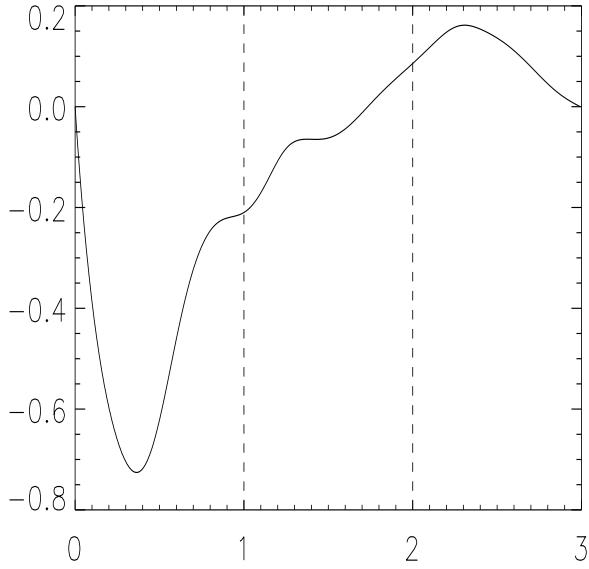
In the context of A-type stars, the conductivity in the convectively stable layer is not expected to vary greatly from that in the unstable zones and so, in the language of the model, the stiffness is low. However, from a mathematical view point it is interesting to give some consideration to what happens if the stiffness of the mid-region is increased. Typical values of the stiffness in the convectively stable layer in previous numerical simulations have reached 15 (Tobias *et al.* (1998); Tobias *et al.* (2001)), a very large value. However, here we choose to push up even further to a  $S_2$  value of 30; this is certainly greater than that which is



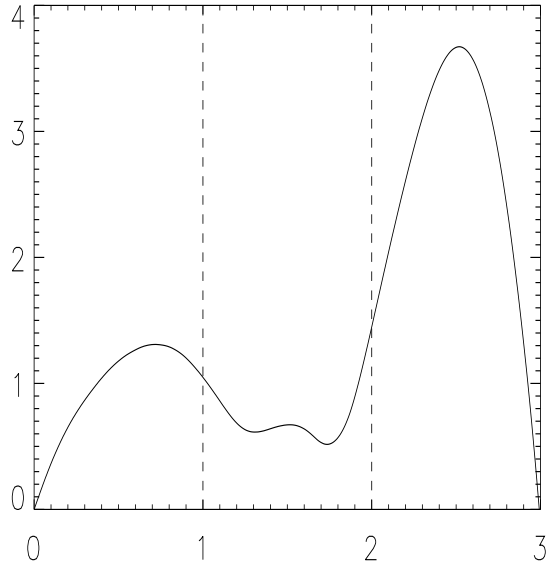
**Figure 5.** Slices showing the vertical component of momentum through each of the three zones.

encountered in A-type stars. Even such a large stiffness parameter only yields a variation of 1.91 pressure scale heights across the stable region. Therefore, the analytic arguments by Latour *et al.* lead us to expect that, even for such an extreme choice of the stiffness parameter, there will be some connection between the two convectively unstable layers.

To show that this conjecture is valid, we fix the height of the box to 3.0 units and consider three further values of  $S_2$  namely, 10 15, and 30. Such values of the mid-layer



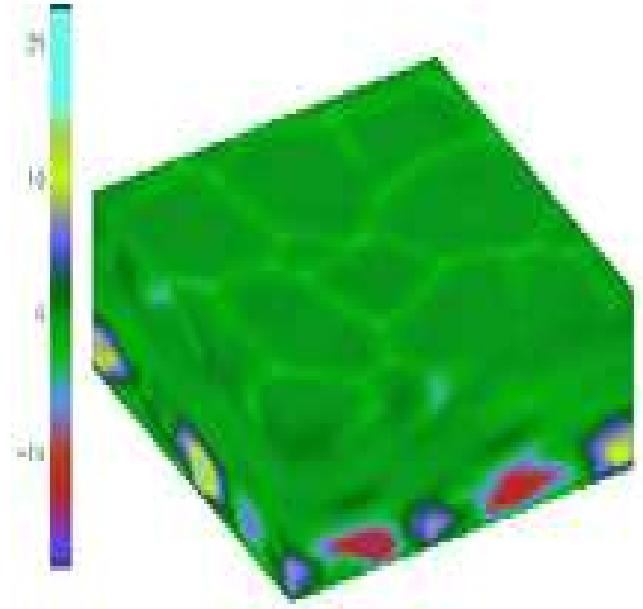
**Figure 6.** Plot at  $t = 15.02$ . The variation of the vertical component of velocity as a function of height at the centre of the box for the case where  $\mathcal{D} = 3.0$  and  $S_2 = 5.0$ .



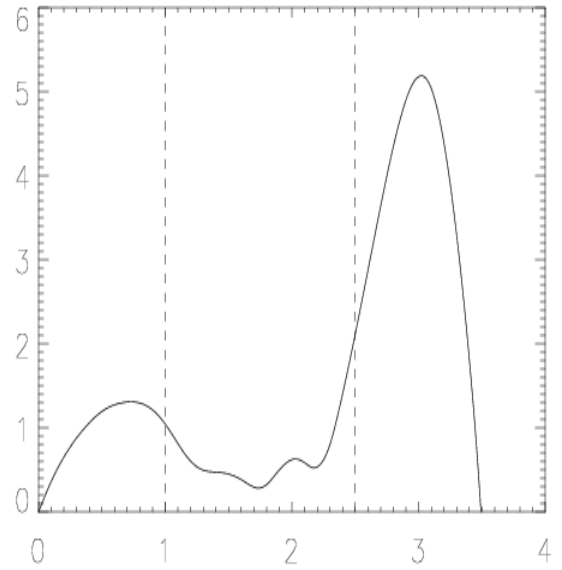
**Figure 7.** Variation of the horizontal average of the modulus of the z-component of momentum at  $t = 15.02$  for the case where  $\mathcal{D} = 3.0$  and  $S_2 = 5.0$ .

stiffness give rise to a difference across the mid-layer of 1.72, 1.78 and 1.91 pressure scale heights.

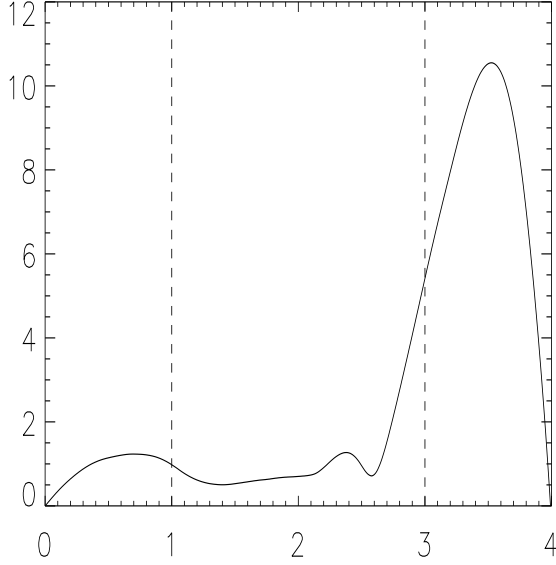
Figures 12-15 clearly show that, even for cases with an extreme mid-layer stiffness, the convectively unstable regions can not satisfactorily be considered as separate entities, which once again has important implications for future models that aim to examine mixing and transport in stars with multiple convection zones.



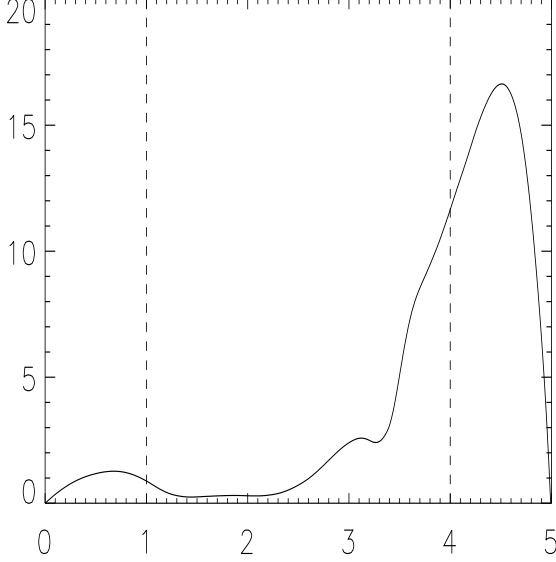
**Figure 8.** Plot at  $t = 15.34$  Sides of the box show the vertical momentum flux and the top of the box shows the vertical momentum flux near the top of the box for the case where  $\mathcal{D} = 3.5$ .



**Figure 9.** Variation of the horizontal average of the modulus of the z-component of momentum at  $t = 15.34$  when  $\mathcal{D} = 3.5$ .



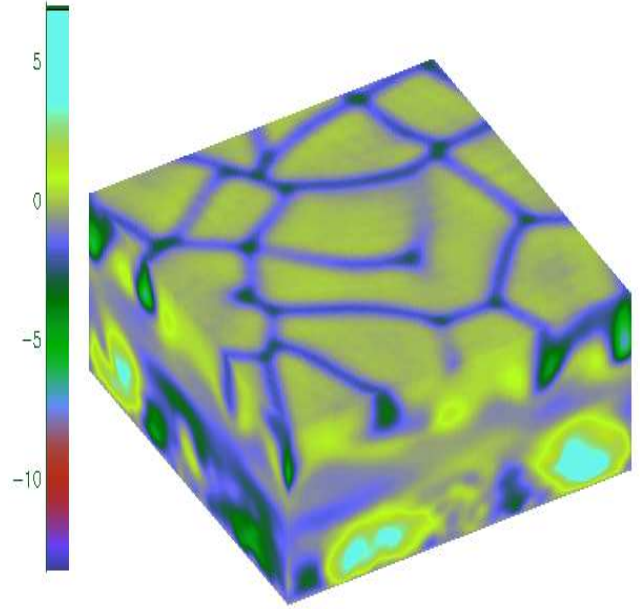
**Figure 10.** Variation of the horizontal average of the modulus of the z-component of momentum at  $t = 15.17$  when  $\mathcal{D} = 4.0$ .



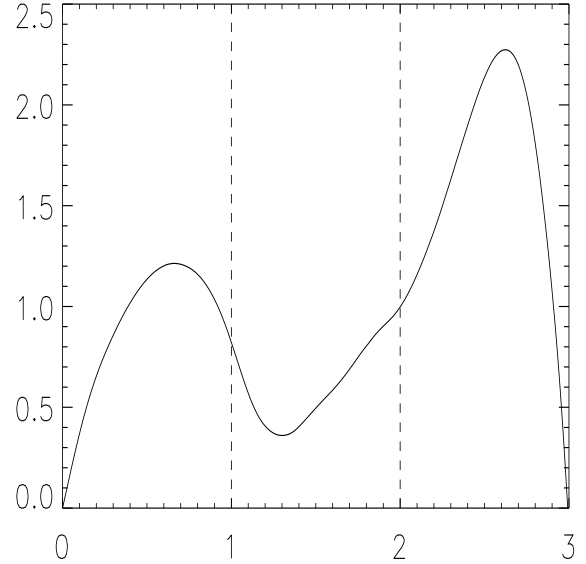
**Figure 11.** Variation of the horizontal average of the modulus of the z-component of momentum at  $t = 15.16$  when  $\mathcal{D} = 5.0$ .

## 5 CONCLUSIONS

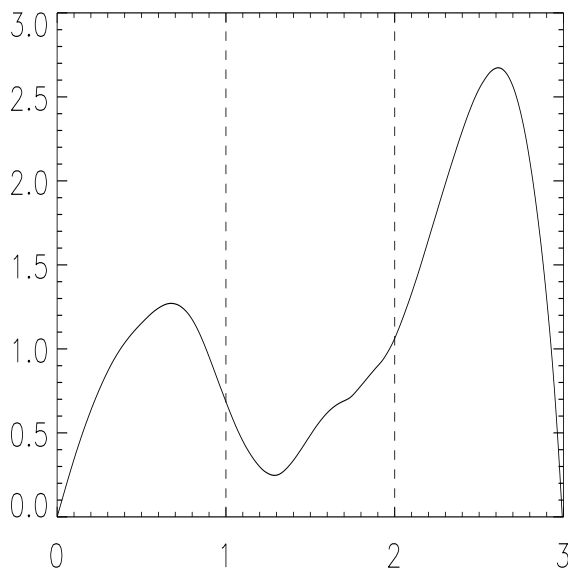
There is still a variety of questions and issues that need to be fully resolved about the complex dynamical interactions that occur within A-type stars. The mere presence of multiple convection zones within these stars implies an extra degree of complexity compared to that found in G-type stars such as the Sun. In the present paper we focus on a basic question concerning the conditions that would be required for the convection zones in A-type stars to be considered as non-interacting. To this end we considered an idealized model to



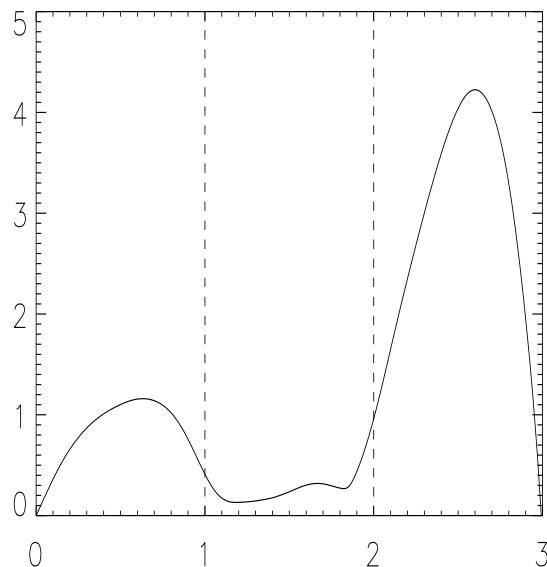
**Figure 12.** Plot at  $t = 15.12$  Sides of the box show the vertical momentum flux and the top of the box shows the vertical momentum flux near the top of the box for the case where  $\mathcal{D} = 3.0$  and  $S_2 = 10$ .



**Figure 13.** Variation of the horizontal average of the modulus of the z-component of momentum at  $t = 15.12$  in the case where  $\mathcal{D} = 3.0$  and  $S = 10$ .



**Figure 14.** Variation of the horizontal average of the modulus of the  $z$ -component of momentum at  $t = 15.17$  for the case where  $S = 15$ .



**Figure 15.** Variation of the horizontal average of the modulus of the  $z$ -component of momentum at  $t = 15.16$  for the case where  $S = 30$ .

shed greater light on the outer convection zones that exist below the stellar photosphere.

In this paper we focused on two potential ways by which the interaction between two convection zones could be reduced, namely the effect of increasing the distance between the two convection zones and increasing the stiffness on the mid-layer. Both of these approaches naturally give rise to an increase in the number of pressure scale heights of variation across the convectively stable layer that separates the

two convectively stable zones, which was suggested to be an important factor in earlier analytic work.

In section 3 we examined the effect of increasing the height of the box from a fiducial case where the box height is such that both convectively unstable and stable regions have the same height. As we increased the height of the box we fixed the height of the unstable layers and therefore a box height increase implies an increase in the height of the convectively stable layer. Such an increase naturally drives up the pressure scale height variation across the stable zone in the middle of the box and so, with this approach, we are also able to test the earlier theory of Latour *et al.* that at least two pressure scale heights of separation is required for the convectively unstable layers not to interact. We showed that even for a box height of five units and a corresponding pressure variation of almost four scale heights we did not find that the convectively unstable regions can be considered as separate, which is not what was suggested by the earlier analytic theory, though quantitative comparison cannot be expected from such idealized models. For main-sequence A-type stars it is unlikely that there are more than four pressure scale heights of difference spanning the convectively stable region that separates the outer two convection zones. Therefore, this work shows that these two regions will interact and the interaction will give rise to drastically different mixing and transport than if they could be considered as separate. It is thus clear that the convection work that was motivated by a desire to understand transport in the solar convection zone does not naturally extend to these stars.

In the second results section we attempted to separate the two convection zones by increasing the stiffness of the stable layer. However, we greatly exceeded the level of stiffness that can be anticipated in the region in A-type stars and we still found that the two convectively stable regions are connected. This adds further weight to the point made above that when contemplating the mixing and transport in A-type stars the convection layers cannot be treated as isolated.

One of the major objectives of this paper is to provide a solid hydrodynamical basis on which more complex models can be constructed to understand fully the dynamics that occur below the surface of A-type stars. With such a simple model we are not yet in a position to address important secondary questions such as the influence of the observed chemical anomalies (see discussions in Michaud (1970) and Vauclair & Vauclair (1982) for more details). However, our model provides a platform on which we can build, so as to address the effects of rotation, magnetic fields and other issues related to the dynamics in these stars. We acknowledge here that it has been suggested by some models that the second convection zone (see, for example, Vauclair, Vauclair & Pamjatnikh (1974) and Richer, Michaud & Proffitt (1992)) could vanish under certain conditions although there is clearly no consensus in the literature. We anticipate that the model could be extended so as to clarify this issue also. Work on these extensions is currently in progress.



## ACKNOWLEDGMENTS

LJS wishes to thank the Department of Applied Mathematics and Theoretical Physics at the University of Cambridge for the award of a Crighton fellowship for partial support of this research. She would also like to acknowledge the financial assistance she received via the Chaire d'Excellence award to Professor Steve Balbus at the Ecole Normale Supérieure in Paris. We thank Steve Houghton for his earlier contributions to the numerical code, Douglas Gough for his helpful comments in the early stages of this work, and Paul Bushby for many useful discussions. Finally, we wish to thank the referee for helpful and constructive comments.

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